Code Cracking the Cube: Comparing Heuristic Rubik's Solution Methods by Programmed Algorithms and Move Efficiency *

By Emily Lamb

Abstract – You've seen it before — that puzzle of colorful squares, the Rubik's cube. As its inventor discovered, a few turns on the cube can make it very difficult to solve. Nevertheless, there are several methods some people have discovered to solve it, but which one is the best? This experiment measures which method uses the fewest number of turns on the cube. I programmed a simulation of the cube and several methods to solve it: two popular beginner methods, a method favored by competitive speedcubers, and some methods I developed myself. I used a set of different scrambles for each experimental trial. Then for each method I measured the number of turns required for each trial and calculated the number of sequences of turns that a person would have to memorize (or that are required by a computer program) to perform the method. My hypothesis was that the more sequences of turns one had to know for a method, the fewer number of turns it takes to solve the cube. The collected results generally supported my hypothesis, as the methods with the fewest number of turns to solve the cube required knowledge of three to 20 times more sequences of turns than other methods. This is an example of heuristic problem solving: knowing more good solutions to small problems, like steps to solve patterns in the cube, helps to build a better solution to a large problem, like the whole cube. It also demonstrates a computer's usefulness in collecting scientific data.

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QUESTION

Which of several popular methods solves the Rubik's cube in the fewest moves?

VARIABLES

Independent – Methods

Dependent – Moves to solution per trial per method, algorithms required per method

Controlled – Turn speed, type of cube (3x3x3 dimension), programming environment

HYPOTHESIS

If I use a method that consists of a greater number of algorithms to be memorized, then the Rubik's cube will be solved in fewer moves.

BACKGROUND RESEARCH

A move is a turn of any angle, but in a cube with rotating sides, a move is typically +/-90 degrees (a quarter turn) or 180 degrees (a half turn). An algorithm is a sequence of moves, such as turn the left (green) face counterclockwise (-) 90 degrees, then turn the front (red) face 180 degrees, and then turn the left (green) face clockwise (+) 90 degrees. A Rubik's cube is a twisty puzzle with 54 colored squares comprising six sides of nine squares each; the puzzle is solved when each side contains only one color. A corner piece is a moveable piece with three different colored faces that remains in a corner of any side. An edge piece is a moveable piece with two different colored faces that remains on an edge of any side. A center piece is a fixed piece with one face that remains in the center of a given side. A speedcuber is one who solves the cube for the fastest time possible; the World Cube Association began in 2004 to host official competitions for the sport of speedcubing.

An architect named Erno Rubik invented the Rubik's cube in 1974. He lived in Budapest, Hungary. He wanted to explain visually to his students about spatial relationships. He first built it out of wood and rubber bands, calling it the "Magic Cube". He started toying with his cube, and then he realized that it did not take long before the pieces were all scrambled up. He started to worry it was unsolvable. Nevertheless, a month later, he managed to solve it. It was released globally in 1980 as a puzzle called the "Rubik's Cube" in honor of its inventor. Since then, many versions of the cube have been introduced; in 2009 Rubik's came out with a cube that responds to touch, and in 2013 Rubik's came out with a stickerless cube as well as a speed cube.

Many analyses have been performed on the Rubik's cube. Some found that there are 43,253,003,274,489,856,000 (43 quintillion) different configurations of the cube. In July of 2010, one study found that any configuration of the 3x3x3 cube can be solved in 20 moves or less. The most common studies work to find which methods are faster than others, based on either the number of seconds or the number of moves it takes for the Rubik's cube to be solved. For example, the Fridrich method, also known as the CFOP method (an acronym for the steps followed in the solution), is the most popular method among speedcubers. The full

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method averages around 56 moves[1], but one must memorize 72 algorithms. The beginner method is very popular as well, especially among beginner cubers, since it is easy to learn. One only has to memorize around eight algorithms, but the method averages around 110 moves[2].

How does the Rubik's cube work? A 3x3x3 cube has 21 pieces. Six squares are attached to the core of the cube, creating the center pieces. They can spin, but they cannot change positions on the cube. Therefore, these all make up one piece. On a traditional cube, the red center will always be opposite the orange, the yellow opposite the white, and the blue opposite the green, with sides ordered red, green, orange, and blue, clockwise if looking at the white side. The rest of the cube is made up of edge and corner pieces. There are eight corner pieces and twelve edge pieces. These pieces are snapped to each other and rotate around the circular cavity of each face. Each piece is usually made of plastic, traditionally black, with colored stickers adhered to it.

What are some popular methods used to solve the Rubik's cube? As stated before, the Fridrich method is the most popular method. It was named for a woman named Jessica Fridrich, who had developed this method and published it in 1997. It is an intermediate method. Her method grew in popularity because of its speed and logical progression that has some similarities to a beginner method. Beginner methods are also popular. They are not very fast, so competitive speedcubers do not use them. However, they are very flexible and easy to learn, so just about everyone who wants to learn to solve the Rubik's cube starts by learning a beginner method. There are many beginner methods. Dan Brown's beginner method is one of the most popular because of its use of algorithms for each step. Another beginner method is the Rob's World beginner method. It uses few algorithms but is very efficient, as it provides a beginner version of some of the more advanced methods like Fridrch. Based on these methods, I have developed one beginner method and two intermediate methods myself to solve the cube. I analyze each of these six methods in my experiment, along with a seventh, which is considered an advanced method. This advanced method was created by a Polish man named Zbigniew Zborowski in 2006, making it the only new method of the 21st century.

What is useful about the Rubik's cube? It builds problem-solving skills, spatial reasoning, cognitive thinking, and even confidence. It demonstrates spatial relationships, so teachers can use it as an example of solid geometry. It can teach young children about the 90- and 180-degree angles. It could even give a history lesson, since it is an object from decades ago; as of 2014, it is 40 years old. People who enjoy learning new things and tackling challenges can have fun solving this complex puzzle. Finally, it is a toy; it entertains people of all ages and gives them something with which to engage their minds.

Materials

Physical 3x3x3 Rubik's cube that has official colors in the official arrangement

Microsoft Visual Basic 2010 Express software

Computer compatible with the Visual Basic software

Experimental Procedure

1. Ensure Visual Basic is installed on computer.

2. Using Visual Basic, create Rubik's cube simulation to represent all six faces (3x3 colored matrices) and 27 different moves (left/right/middle planes, of top/front/right [or bottom/back/left, etc.] faces, rotated clockwise/counterclockwise/180-degrees).

3. Test the program with several scrambles to make sure it works to this point correctly. If not, fix the program and repeat this step.

4. Add programming to report the number of moves taken and to check the simulation after each move to determine whether the cube is solved.

5. Repeat step 3.

6. Create an event in Visual Basic's Design window to scramble or apply solution moves to the cube. The event could be used to synchronize moves at a given speed or just to trigger proper execution of the methods in the graphical window.

7. Repeat step 3.

8. Using Visual Basic's pseudo-random number generator with a known starting seed value (for repeatability across methods), scramble the Rubik's cube simulation with a sequence of approximately 30 moves.

9. Repeat step 3.

10. Scramble the physical cube and test one method on it. Repeat with various scrambles.

11. If the method works as expected every time, then program the method to solve the scrambled cube simulation.

12. Repeat step 3. The program now can replace the human for this method, working faster and producing more results with consistent behavior.

[1] "How to Speedsolve the Rubik's Cube"

[2] *Ibid.*

13. Count how many algorithms (move sequences) were required to program that method (i.e., how many such sequences a human would have to memorize to perform it).

14. Repeat steps 8 and 9 six times to create six different cube scramble sequences (random seeds). Also use six example scrambles from literature to enable comparison to other methods not programmed and reveal further program issues in testing because of intentionally difficult or degenerate cube configurations.

15. Repeat steps 10-13 to create the six different methods to be compared (by using every cube scramble sequence with each method).

Data Analysis

Dan Brown Beginner's Method averaged 170 moves and consisted of nine algorithms. This implementation is based on a widely popular introduction to solving the Rubik's cube.

Rob's World Beginner's Method averaged 149 moves and consisted of nine algorithms. (This may indicate a method can be generally superior based on the sequences used and their order and not just the number of sequences.) This implementation is based on the Rob's World YouTube channel approach to the beginner's method.

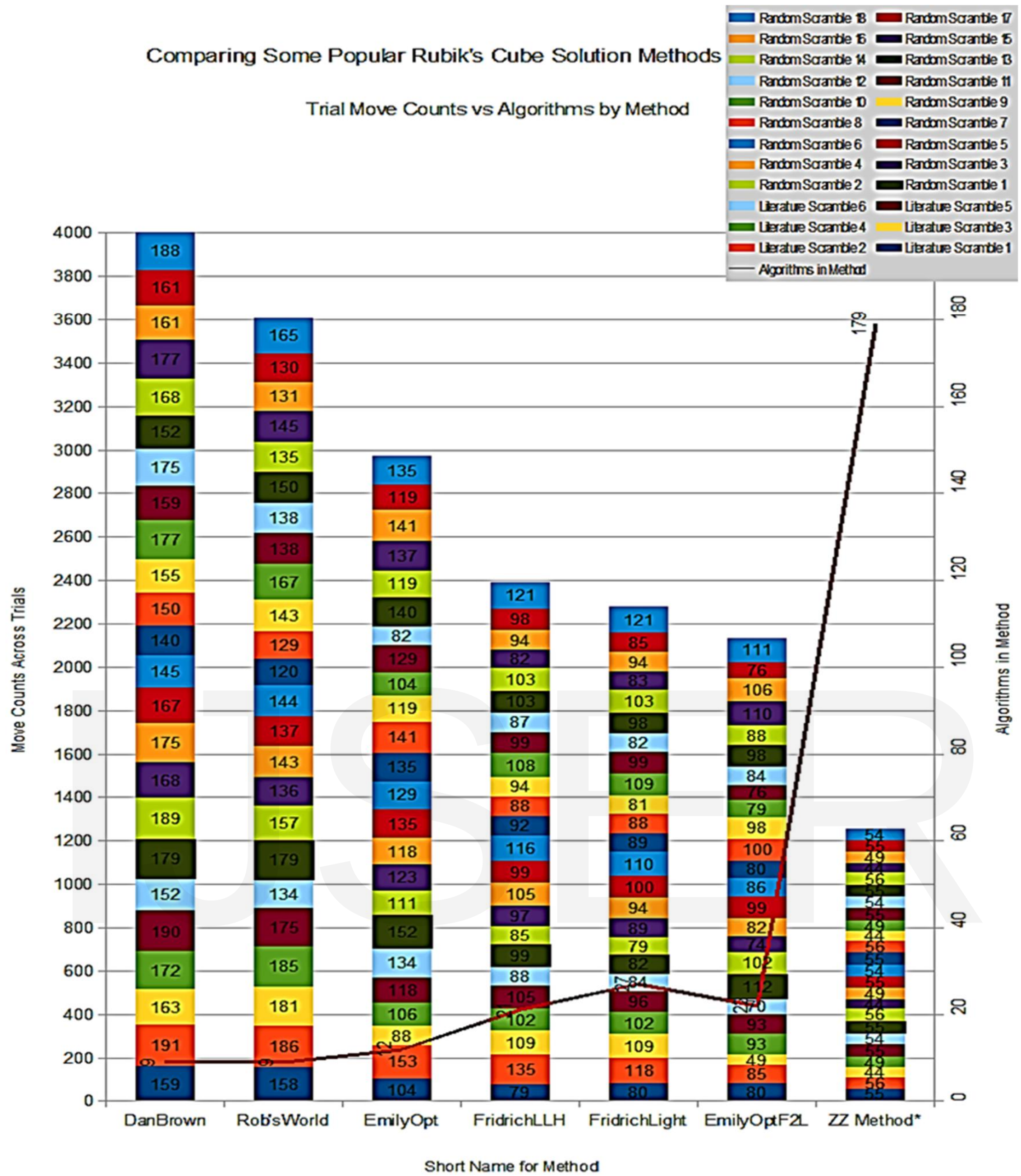
Emily's Optimized Rob's World Method averaged 128 moves and consisted of 12 algorithms. This method used an advanced cross technique at the beginning and a minimal hybrid set of last-layer algorithms alternating position and orientation strategies derived from other methods. Since this author has not found such an algorithm described elsewhere in literature, it is distinguished by the author's name from the beginner's method on which it was built. It has a higher number of sequences and lower move counts than the other beginner methods.

Fridrich Method averaged 92 moves and consisted of 27 algorithms. Note the implementation used could be called "Fridrich Light" as it used a smaller number of algorithms because of simplified "two-look last-layer" sequences instead of numerous "one-look last-layer" sequences often associated with Fridrich. This can explain the lower number of sequences and higher move counts than found in literature under Fridrich. This intermediate method has a higher number of sequences and lower move counts than all the beginner's methods (and some Fridrich results in the literature indicate an even higher number of move sequences and even lower move counts).

Emily's Optimized Rob's World Method with F2L averaged 93 moves (over the six random scrambles, or lower when using all 24 scrambles including an outlying low result) and consisted of 20 algorithms. This hybrid method adopted the simultaneous first two layers step (called F2L) of the Fridrich method with the other enhancements to the beginner method described earlier under the Emily's Optimized Rob's World Method. This yielded a higher number of move sequences than the original Emily's Optimized method but a lower number of move sequences than the Fridrich implementation and correspondingly lower move counts than the original Emily's Optimized method but comparable move counts to Fridrich.

Fridrich Method with Last Layer Hybrid Sub-Method averaged 101 moves and consisted of 21 algorithms. The substitution of an even less advanced set of last-layer algorithms reduced the number of move sequences but correspondingly increased the move counts. It lacks the advanced cross technique of the F2L variant of Emily's Optimized method but is otherwise identical.

The ZZ Method is a very fast method, averaging 52 moves. However, it requires 179 algorithms, making it a very advanced method. I could not program this method, so I found a website (cube.crider.co.uk) and used its example solves for comparison to the other methods that I did program.



*Uses 6 literature example solves 4 times

Conclusions

The results generally support my hypothesis. As the number of move sequences increased, the move counts observed decreased.

The Fridrich method had the most algorithms and used the fewest moves. However, Emily's Optimized Rob's World Method with F2L ("EORWF") used seven fewer algorithms than the Fridrich method, yet it used only one move more than the Fridrich method.

EORWF used the second-greatest number of moves, but the method with the third-greatest number of moves, the Fridrich Method with Last Layer Hybrid Sub-Method, consisted of one more algorithm than EORWF, despite its move count being much greater than that of EORWF's. The trend was preserved, but the choice of move sequences and their ordering appears to play a significant role in the move counts as well.

Given the measurable difference between the Dan Brown and Rob's World beginner methods implementations, despite their equal number of move sequences, however, there must be another factor in addition to the number of move sequences that determined how many moves were required. That would be an interesting area for further study. In addition, there are other methods less dependent on memorized move sequences and more dependent on in-depth experience ("acquired intuition," one might say) in solving smaller parts (e.g., certain 2x2x2 sub-blocks) of the cube that cannot be analyzed by a simple count of move sequences. Quantifying the complexity of such methods would be another interesting area of study.

My experimental procedure worked well, but it took a lot of time to diagnose and fix errors in my code. Although creating a working program was slow, the computer ended up solving the cube faster than any speedcuber could, and it never made mistakes caused by human error. This is exactly the reason I chose to test my hypothesis using computer programming rather than working it out by hand. Testing my hypothesis this way also had an effect on me that I did not foresee: it taught me more about the Rubik's cube and programming than I've ever learned from any book, article, or video.

Because this experiment was so time-consuming, I could not program more than three basic to intermediate methods and three derived methods. For further study, I could learn more about the other methods I could not get to for this experiment.

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